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$D = 5$ SU(2) $\times$ U(1) gauged supergravity from $D = 11$ supergravity

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Abstract: We consider the most general class of supersymmetric solutions of $D = 11$ supergravity consisting of a warped product of AdS_5 with a six-dimensional internal manifold $\mathcal{N}_6$, which are dual to $N = 2$ super conformal field theories in $d = 4$. For any such $\mathcal{N}_6$ we construct the full non-linear Kaluza-Klein ansatz for the reduction of $D = 11$ supergravity on $\mathcal{N}_6$ down to $D = 5$ SU(2) $\times$ U(1) gauged supergravity, at the level of the bosonic fields. This allows one to uplift any solution of the $D = 5$ supergravity to obtain a solution of $D = 11$ supergravity for any given $\mathcal{N}_6$. Using an explicit $\mathcal{N}_6$, corresponding to M5-branes wrapping holomorphic curves in a Calabi-Yau two-fold, we uplift some solutions and comment upon their interpretation.

Keywords: AdS-CFT Correspondence, Supergravity Models.

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1. Introduction

A Kaluza-Klein (KK) reduction of a higher dimensional theory of gravity down to a lower dimensional theory using an internal manifold $\mathcal{N}$ is said to be consistent if any solution to the equations of motion of the lower-dimensional theory can be uplifted on $\mathcal{N}$ to obtain a solution to the equations of motion of the higher-dimensional theory. If one keeps the entire infinite tower of KK modes the reduction is obviously consistent, since the reduction is simply a rewriting of the original higher dimensional theory. However, in certain special cases it is possible to obtain consistent KK reductions by further truncating to a finite number of modes.

The standard way to prove that a KK reduction is consistent is to construct a KK ansatz: i.e. an explicit embedding of the fields of the lower-dimensional theory into the higher-dimensional one, with the property that the equations of motion of the higher-dimensional theory are satisfied provided that the equations of motion of the lower-dimensional theory are. Such a KK ansatz has the virtue that any explicit solution of the lower-dimen-sional theory can be uplifted to obtain an explicit solution of the higher dimensional theory. This has proved to be a very powerful technique for constructing supergravity solutions relevant for string/M-theory.

While much is now known about consistent KK truncations a single overarching principle governing all cases remains elusive, if indeed one exists. Recently we put forward a conjecture [1] (related to [2]) for sufficient criteria for consistency, covering a large number of supersymmetric cases. Consider the most general supersymmetric solutions of $D = 10$ or $D = 11$ supergravity[1] that are (warped) products, $AdS_{d+1} \times_w \mathcal{N}$, of a $d+1$-dimensional anti-de-Sitter space, AdS_{d+1}, with an internal space, $\mathcal{N}$, that are dual to supersymmetric conformal field theories (SCFTs) in d dimensions. We conjectured that there should be a consistent KK reduction on $\mathcal{N}$ to a gauged supergravity theory in $d+1$ dimensions for which the fields are dual to those in the superconformal current multiplet of the d-dimensional dual SCFT.

We now know that this conjecture is in fact a theorem for a number of different cases for which the explicit KK reduction ansatze have been constructed. For example, for the class of $AdS_5 \times M_5$ solutions of type IIB supergravity, where M_5 is a Sasaki-Einstein manifold, that are dual to $N = 1$ SCFTs in $d = 4$, it was shown in [3] (see also [4]) that there is a consistent KK reduction of IIB supergravity on M_5 to minimal gauged supergravity in $D = 5$. This result was generalised in [1] to the most general class of $AdS_5 \times_w \mathcal{N}_5$ solutions of type IIB that are dual to $N = 1$ SCFTs in $d = 4$ [5], thus verifying the conjecture in this case. It was shown in [6] that minimal $D = 5$ gauged supergravity also arises from the KK reduction of $D = 11$ supergravity on $\mathcal{N}_6$ associated with the most general class of $AdS_5 \times_w \mathcal{N}_6$ solutions of $D = 11$ with $d = 4$, $N = 1$ SCFT duals [7]. Furthermore, [6] also showed that for general classes of $AdS_4 \times_w \mathcal{N}_7$ solutions that are dual to $N = 2$ SCFTs in $d = 3$ there is a consistent KK reduction on $\mathcal{N}_7$ to $N = 2$ gauged supergravity in $D = 4$. Of course, the well-known consistent KK reductions of $D = 11$ supergravity on S^4 [8] and S^7 [9, 10], or of IIB supergravity[2] on S^5 (see, e.g., [11–14]), related to the maximally supersymmetric solutions $AdS_7 \times S^4$, $AdS_4 \times S^7$ and $AdS_5 \times S^5$, respectively, are also examples supporting the conjecture.

In this paper we will consider the general class of $AdS_5 \times_w \mathcal{N}_6$ solutions of $D = 11$ supergravity that are dual to $N = 2$ SCFTs in $d = 4$. Such supergravity solutions were classified by Lin, Lunin and Maldacena in [15], refining the work of [7]. Such SCFTs have an SU(2) × U(1) R-symmetry and so the conjecture of [1] says that there should be a consistent KK reduction of $D = 11$ supergravity on $\mathcal{N}_6$ to Romans' $D = 5$ SU(2) × U(1) gauged supergravity [16] (more precisely to what is called the $N = 4^+$ theory in [16]). In this paper we will construct the full non-linear KK ansatz for the bosonic fields.

At a technical level, this case is considerably more involved than the previous cases considered in [6, 1]. The central subtlety in guessing the correct KK ansatz is the correct incorporation of the scalar field of the $D = 5$ gauged supergravity. We found the results of [14, 17] to be particularly helpful. In [14] the full KK ansatz for the reduction of type IIB supergravity on an S^5 to Romans' theory was presented. This is expected to

[1] One can also consider supergravity theories in other dimensions.

[2] No complete reduction ansatz of IIB supergravity on S^5 has been yet constructed.

be a truncation of a more general KK ansatz to maximally supersymmetric SO(6) gauged supergravity. Now, after T-duality and uplifting, it is known [18] that the $AdS_5 \times S^5$ solution of type IIB supergravity can be used to obtain the singular $AdS_5 \times_w \mathcal{N}_6$ solution of $D = 11$ supergravity found in [19]. By performing the same T-duality and uplifting on the type IIB KK reduction ansatz for the S^5 found in [14], it was shown in [17] how one can obtain Romans' theory by reduction of the specific $D = 11$ solution found in [19]. The form of the KK reduction ansatz, for this specific solution, provided us with important clues in obtaining the ansatz for an arbitrary $AdS_5 \times_w \mathcal{N}_6$ solution that we shall present here.

The only regular $AdS_5 \times_w \mathcal{N}_6$ solution that we are aware of is the solution constructed by Maldacena and Núñez in [20]. This solution is dual to the $N = 2$ SCFT in $d = 4$ that lives on $M5$-branes wrapping holomorphic Riemann surfaces in Calabi-Yau two-folds. Using this solution we can uplift any explicit solution of Romans' theory to obtain an explicit solution of $D = 11$ supergravity. We uplift some known solutions of Romans' theory and discuss how some of them are related to wrapped brane solutions.

The calculations required for checking that our KK ansatz is correct are quite involved and so we have included a few details in an appendix.

2. Romans from Lin, Lunin and Maldacena via Kaluza and Klein

In this section we present the KK ansatz for the reduction of $D = 11$ supergravity on the geometries $\mathcal{N}_6$ classified by Lin, Lunin and Maldacena (LLM), down to Romans' $D = 5$ SU(2) $\times$ U(1) gauged supergravity. We begin by first reviewing the work of LLM and Romans in subsections 2.1 and 2.2, respectively.

2.1 The geometry of LLM

The geometry underlying the most general AdS_5 solutions of $D = 11$ supergravity that are dual to $N = 2$ SCFTs in $d = 4$ was first derived by LLM in [15], where it was shown that such supergravity solutions are determined by solutions of a continuous three dimensional version of the Toda equation. The same conditions were rederived from a different point of view in [21], by taking the AdS limit of a general class of Minkowski geometries corresponding to M5-branes wrapped on a Kähler two-cycle in a Calabi-Yau two-fold. We will use the notation of [21], which also includes the explicit dictionary between the two descriptions. Our conventions for $D = 11$ supergravity are as in [22], some of which is recorded in appendix A.

The metric is a warped product of AdS_5, with radius $1/m$, with a six-dimensional internal manifold $\mathcal{N}_6$:

$$\mathrm{d}s^2_{11} = \lambda^{-1}\mathrm{d}s^2(AdS_5) + \mathrm{d}s^2(\mathcal{N}_6), \tag{2.1}$$

where the warp factor λ is a function of the coordinates on $\mathcal{N}_6$ only. As in [21], we will let $(e^1, \ldots, e^6)$ be an orthonormal frame for $\mathcal{N}_6$,

$$\mathrm{d}s^2(\mathcal{N}_6) = (e^1)^2 + (e^2)^2 + (e^3)^2 + (e^4)^2 + (e^5)^2 + (e^6)^2 \ , \tag{2.2}$$

with

$$
\begin{aligned}
e^4 &= \frac{\lambda}{2m\sqrt{1-z}}\mathrm{d}\rho \\
(e^5)^2 + (e^6)^2 &= \frac{\lambda^2\rho^2}{4m^2}\mathrm{d}\mu^i\mathrm{d}\mu^i \,.
\end{aligned} \tag{2.3}
$$

Here, ρ is a coordinate, $z \equiv \lambda^3\rho^2$ and the μ^i, $i = 1, 2, 3$, satisfying $\mu^i\mu^i = 1$, parametrise a two-sphere. Note that the one-form $\hat{\rho}$ in [21] is denoted here by e^4. We will define a positive orientation by $\epsilon = e^{123456}$. The four-form flux is given by

$$
G_4 = -\frac{1}{8m^2}\epsilon_{ijk}\mu^i\mathrm{d}\mu^j \wedge \mathrm{d}\mu^k \wedge \left[\mathrm{d}\left(\lambda^{1/2}\rho\sqrt{1-z}e^3\right) + 2m\left(\lambda\rho e^{12} + \lambda^{-1/2}e^{34}\right)\right]. \tag{2.4}
$$

The necessary and sufficient conditions for (2.1), (2.4) to be a supersymmetric solution to the equations of motion of $D = 11$ supergravity are

$$
\begin{aligned}
\mathrm{d}\left(\lambda^{-1}\sqrt{1-z}e^1\right) &= m\lambda^{-1/2}\left(\lambda^{3/2}\rho e^{14} + e^{23}\right), \\
\mathrm{d}\left(\lambda^{-1}\sqrt{1-z}e^2\right) &= m\lambda^{-1/2}\left(\lambda^{3/2}\rho e^{24} - e^{13}\right), \\
\mathrm{d}\left(\frac{\lambda^{1/2}}{\sqrt{1-z}}e^3\right) &= -\frac{2m\lambda}{1-z}e^{12} - \frac{3\lambda\rho}{(1-z)^{3/2}}\left[(\mathrm{d}\lambda)_4 e^{12} - (\mathrm{d}\lambda)_2 e^{14} + (\mathrm{d}\lambda)_1 e^{24}\right],
\end{aligned} \tag{2.5}
$$

where

$$
\mathrm{d}\lambda = (\mathrm{d}\lambda)_1 e^1 + (\mathrm{d}\lambda)_2 e^2 + (\mathrm{d}\lambda)_4 e^4 \,, \tag{2.6}
$$

and $e^{12} \equiv e^1 \wedge e^2$, etc.

The $d = 4$, $N = 2$ dual SCFTs have an $\mathrm{SU}(2) \times \mathrm{U}(1)$ R-symmetry, and this manifests itself as isometries of the internal metric $\mathrm{d}s^2(\mathcal{N}_6)$. The $\mathrm{SU}(2)$ symmetry of the two-sphere, parametrised by the μ^i, is clearly a symmetry of the solution. The vector field dual to e^3 is proportional to the additional $\mathrm{U}(1)$ Killing vector, consistent with (2.6). This is explained in more detail in [21] where it is shown how the above conditions allow one to introduce coordinates, used by [15], which makes the $\mathrm{U}(1)$ symmetry manifest. One subtlety is that the frame we will be using depends on the coordinate parametrising the orbits of this $\mathrm{U}(1)$ and so, for example, we have from (2.5)

$$
\begin{aligned}
(\mathrm{d}(\mathrm{d}\lambda)_1)_3 &= -\frac{m\lambda^{1/2}}{\sqrt{1-z}}(\mathrm{d}\lambda)_2 \,, \\
(\mathrm{d}(\mathrm{d}\lambda)_2)_3 &= \frac{m\lambda^{1/2}}{\sqrt{1-z}}(\mathrm{d}\lambda)_1 \,.
\end{aligned} \tag{2.7}
$$

2.2 Romans' $D = 5$ $\mathrm{SU}(2) \times \mathrm{U}(1)$ supergravity

The field content of Romans' ($N = 4^+$) $\mathrm{SU}(2) \times \mathrm{U}(1)$ gauged supergravity in $D = 5$ [16] consists of a metric, with line element $\mathrm{d}s_5^2$, a scalar field X, $\mathrm{U}(1) \times \mathrm{SU}(2)$ gauge fields B, A^i, with $i = 1, 2, 3$, and a complex two form C which is charged with respect to the $\mathrm{U}(1)$

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gauge field. The corresponding field strengths for these potentials are given by

$$
\begin{aligned}
G &= \mathrm{d}B, \\
F^i &= \mathrm{d}A^i - \frac{1}{\sqrt{2}} m\epsilon_{ijk} A^j \wedge A^k, \\
F &= \mathrm{d}C + imB \wedge C.
\end{aligned} \tag{2.8}
$$

In our conventions, the equations of motion for the scalar and the gauge fields are

$$
\begin{aligned}
\mathrm{d}(X^{-1} *\mathrm{d}X) &= \frac{1}{3} X^4 *G \wedge G - \frac{1}{6} X^{-2} \left(*F^i \wedge F^i + *C \wedge \bar{C} \right) \\
&\quad - \frac{4}{3} m^2 \left(X^2 - X^{-1} \right) *\mathbb{1},
\end{aligned} \tag{2.9}
$$

$$
\mathrm{d}(X^4 *G) = -\frac{1}{2} F^i \wedge F^i - \frac{1}{2} \bar{C} \wedge C, \tag{2.10}
$$

$$
D(X^{-2} *F^i) = -F^i \wedge G, \tag{2.11}
$$

$$
X^2 *F = i\, m\, C\,, \tag{2.12}
$$

where $D(X^{-2} * F^i) \equiv \mathrm{d}(X^{-2} * F^i) + \sqrt{2} m\epsilon_{ijk} A^k \wedge (X^{-2} * F^j)$, we have taken $\epsilon_{01234} = +1$ for the five-dimensional space, and $\bar{C}$ denotes complex conjugate of C. In addition, the Einstein equation reads

$$
\begin{aligned}
R_{\mu\nu} &= 3X^{-2}\, \partial_\mu X\, \partial_\nu X - \frac{4}{3} m^2 \left(X^2 + 2X^{-1} \right) g_{\mu\nu} \\
&\quad + \frac{1}{2} X^4 \left(G_\mu{}^\rho G_{\nu\rho} - \frac{1}{6} g_{\mu\nu}\, G_{\rho\sigma} G^{\rho\sigma} \right) + \frac{1}{2} X^{-2} \left(F^i_\mu{}^\rho F^i_{\nu\rho} - \frac{1}{6} g_{\mu\nu}\, F^i_{\rho\sigma} F^{i\rho\sigma} \right) \\
&\quad + \frac{1}{2} X^{-2} \left(C_{(\mu}{}^\rho\, \bar{C}_{\nu)\rho} - \frac{1}{6} g_{\mu\nu}\, C_{\rho\sigma} \bar{C}^{\rho\sigma} \right).
\end{aligned} \tag{2.13}
$$

These equations of motion can be derived from the five-dimensional Lagrangian given by

$$
\begin{aligned}
\mathcal{L} &= R *\mathbb{1} - 3X^{-2} *\mathrm{d}X \wedge \mathrm{d}X - \frac{1}{2} X^4 *G \wedge G - \frac{1}{2} X^{-2} \left(*F^i \wedge F^i + *C_{(2)} \wedge \bar{C} \right) \\
&\quad - \frac{i}{2m} C \wedge \bar{F} - \frac{1}{2} F^i \wedge F^i \wedge B + 4m^2 (X^2 + 2X^{-1}) *\mathbb{1}\,.
\end{aligned} \tag{2.14}
$$

Note that the scalar X can be written in terms of a canonically-normalised dilaton ϕ as $X = e^{-\frac{1}{\sqrt{6}}\phi}$. This Lagrangian can be obtained from the one in [16], up to an overall factor, after changing the signature of the metric, taking $g_1 = -2m$, $g_2 = -2\sqrt{2}m$, $\xi = X^{-1}$ and scaling the gauge fields by a factor of 1/2. We also note that if we set $m = -g$ we have exactly the same equations of motion and Lagrangian as given in [14], except that we disagree with the definition of $D(X^{-2} * F^i)$ by a sign.

Finally, for later use, we note that if we restrict to configurations with $X = 1$, $F^1 = F^2 = C = 0$ and $F^3 = \sqrt{2}G$, the equations of motion (2.9)–(2.13) of Romans theory truncate to

$$
\mathrm{d} * G = -G \wedge G \tag{2.15}
$$

$$
R_{\mu\nu} = -4m^2 g_{\mu\nu} + \frac{3}{2} \left(G_{\mu\lambda} G_\nu{}^\lambda - \frac{1}{6} g_{\mu\nu} G_{\lambda\rho} G^{\lambda\rho} \right). \tag{2.16}
$$

These are the equations of motion of minimal $D = 5$ gauged supergravity [23]. In particular, we can use the reduction formulae given in subsection 2.3 to uplift any solution of $D = 5$ minimal gauged supergravity to obtain a solution of $D = 11$ supergravity.

2.3 The Kaluza-Klein ansatz

We are now in a position to construct the full non-linear ansatz for the KK reduction of $D = 11$ supergravity on any of the six-dimensional manifolds $\mathcal{N}_6$ reviewed in subsection 2.1 down to Romans' $D = 5$ SU(2) × U(1) gauged supergravity.

The KK ansatz for the metric takes the form

$$\mathrm{d}s_{11}^2 = X^{-1/3}\Delta^{1/3}\lambda^{-1}\mathrm{d}s_5^2 + \mathrm{d}s^2(\hat{\mathcal{N}}_6)\,, \tag{2.17}$$

where we have introduced the ubiquitous quantity

$$\Delta = Xz + X^{-2}(1-z)\,. \tag{2.18}$$

In addition

$$\mathrm{d}s^2(\hat{\mathcal{N}}_6) = X^{2/3}\Delta^{1/3}\left[(e^1)^2+(e^2)^2+(e^4)^2\right]+X^{5/3}\Delta^{-2/3}(\hat{e}^3)^2+X^{-4/3}\Delta^{-2/3}\frac{\lambda^2\rho^2}{4m^2}D\mu^iD\mu^i \tag{2.19}$$

where

$$\begin{aligned}\hat{e}^3 &= e^3 + \frac{\sqrt{1-z}}{\lambda^{1/2}}B\,, &(2.20)\\ D\mu^i &= \mathrm{d}\mu^i + \sqrt{2}m\epsilon_{ijk}A^k\mu^j\,. &(2.21)\end{aligned}$$

The way that the SU(2) × U(1) gauge fields are incorporated here follows the usual general principles of KK reductions. The way that the scalar field enters is much less obvious as is the expression for the four-form which is given by

$$G_4 = \tilde{G}_4 + G\wedge\beta_2 + F^i\wedge\beta_2^i + *F^i\wedge\beta_1^i + (C\wedge\alpha_2 + F\wedge\alpha_1 + c.c.) \tag{2.22}$$

where here and in the following $*$ is the Hodge dual with respect to the metric $\mathrm{d}s_5^2$, *c.c.* denotes complex conjugation,

$$\begin{aligned}\tilde{G}_4 = &-\frac{1}{8m^2}\epsilon_{ijk}\mu^iD\mu^j\wedge D\mu^k\wedge\Bigg[\mathrm{d}\left(X^{-2}\Delta^{-1}\rho(1-z)\right)\wedge\frac{\lambda^{1/2}}{\sqrt{1-z}}\hat{e}^3\\ &+X^{-2}\Delta^{-1}\rho(1-z)\mathrm{d}\left(\frac{\lambda^{1/2}}{\sqrt{1-z}}e^3\right)+2m\left(\lambda\rho e^{12}+\lambda^{-1/2}\hat{e}^{34}\right)\Bigg]\end{aligned} \tag{2.23}$$

(we emphasise that in the second term there is no hat on the e^3 –the term should be constructed from (2.5)), $\hat{e}^{34} \equiv \hat{e}^3 \wedge e^4$ (and, in general, $\hat{e}^{3a_1\cdots a_k} \equiv \hat{e}^3 \wedge e^{a_1\cdots a_k}$) and

$$
\begin{aligned}
\beta_2 &= \frac{1}{8m^2}\rho z X \Delta^{-1}\epsilon_{ijk}\mu^i D\mu^j \wedge D\mu^k\,,\\
\beta_2^i &= \frac{1}{2\sqrt{2}m}\left[X^{-2}\Delta^{-1}\rho\lambda^{1/2}\sqrt{1-z}D\mu^i\wedge\hat{e}^3 - 2m\mu_i(\lambda\rho e^{12} + \lambda^{-1/2}\hat{e}^{34})\right]\,,\\
\beta_1^i &= -\frac{X^{-2}}{2\sqrt{2}m}\left(\mu^i \mathrm{d}\rho + \rho D\mu^i\right)\,,\\
\alpha_1 &= \frac{1}{2\sqrt{2}m}\lambda^{-1}\sqrt{1-z}(e^1 - ie^2)\,,\\
\alpha_2 &= \frac{1}{2\sqrt{2}}(e^1 - ie^2)\wedge\left(\lambda\rho e^4 + i\lambda^{-1/2}\hat{e}^3\right)\,.
\end{aligned}
\tag{2.24}
$$

Of course, when the $D = 5$ fields are trivial, $X = 1$, $B = 0$, $A^i = 0$, $C = 0$, the KK reduction ansatz (2.17), (2.22) reduces to the undeformed geometry (2.1), (2.4).

After a lengthy calculation, one can show that the KK reduction ansatz (2.17), (2.22) satisfies the equations of motion of $D = 11$ supergravity (A.1)–(A.3), provided the $D = 5$ fields satisfy the equations of motion (2.9)–(2.13) of Romans theory. This shows, at the level of the bosonic fields, the consistency of the KK reduction. See appendix A for some details of the consistency proof.

3. Uplifting explicit solutions and wrapped branes

The only regular $AdS_5 \times_w \mathcal{N}_6$ solution with $N = 2$ supersymmetry that we are aware of is the solution found by Maldacena and Núñez in [20]. This solution is dual to the $N = 2$ $d = 4$ CFT living on M5-branes wrapping a Riemann surface holomorphically embedded in a Calabi-Yau two-fold. More precisely, the CFT is obtained after taking a decoupling limit and then flowing to the far IR as we will elaborate on a little further below. Using the explicit formulae of subsection 2.3, this solution can be used to uplift explicit solutions of Romans' $D = 5$ $\mathrm{SU}(2)\times\mathrm{U}(1)$ gauged supergravity to obtain explicit solutions of $D = 11$ supergravity.

Setting $m = 1/2$ for simplicity, we find that the $D = 11$ metric (2.17) of the uplifted solution takes now the explicit form

$$
\begin{aligned}
\mathrm{d}s^2_{11} &= 2^{-2/3}\bar{\Delta}^{1/3}\mathrm{d}s^2_5 + 2^{1/3}X\bar{\Delta}^{1/3}[\mathrm{d}\theta^2 + \mathrm{d}s^2(H_2)]\\
&\quad +2^{1/3}X\bar{\Delta}^{-2/3}\sin^2\theta\left(\mathrm{d}x_3 + V + \frac{1}{2}B\right)^2 + 2^{-2/3}X^{-2}\bar{\Delta}^{-2/3}\cos^2\theta D\mu^i D\mu^i
\end{aligned}
\tag{3.1}
$$

where we have replaced, for convenience, the coordinate ρ by a new coordinate θ such that $\rho = \frac{1}{2}\cos\theta$, x_3 is a coordinate along the $\mathrm{U}(1)$ Killing direction e^3, $ds^2(H_2)$ is the standard metric on a unit radius hyperbolic two plane, $dV = -vol(H_2)$ and $\bar{\Delta} = \cos^2\theta + (X^{-3}/2)\sin^2\theta$. We note that here and in the following we can quotient H_2 by a discrete group of isometries, H_2/Γ, and hence obtain a compact Riemann surface with genus greater than one, without breaking supersymmetry. This is the Riemann surface that the M5-branes are wrapping. In obtaining (3.1) we have used the expression for the solution of [20] as given in [15] and then used appendix D of [21] to translate it into the language of this paper. For example, we note that $z = 2\cos^2\theta/(1+\cos^2\theta)$.

In the following subsections, we will use (3.1) to uplift some explicit supersymmetric solutions of Romans' $D = 5$ theory to obtain explicit solutions of $D = 11$ supergravity, some of which are new. We will also discuss how these solutions are related to other solutions in $D = 11$ and IIB supergravity.

3.1 Uplifting the Nieder-Oz solution

Following [20], Nieder and Oz considered the following ansatz for the $D = 5$ supergravity fields [24] (translated into our conventions):

$$\begin{aligned}
ds_5^2 &= \frac{e^{2f}}{4m^2}\left[-dt^2+dr^2\right]+\frac{e^{2g}}{4m^2}ds^2(H_3)\\
X &= e^{-\varphi}\\
A^i &= \frac{1}{2\sqrt{2}m}\epsilon_{ijk}\omega^{jk}\\
B &= 0\,,\quad C=0
\end{aligned}\tag{3.2}$$

where ω^{ij} is the spin connection of the unit radius metric on a three-dimensional hyperboloid H_3 and f, g, φ are functions of r only. Once again, here and in the following, one can replace H_3 with H_3/Γ, whilst preserving supersymmetry. This is indeed a supersymmetric solution providing that f, g, φ satisfy

$$\begin{aligned}
e^{-f}\dot{f} &= -\frac{1}{3}e^{-\varphi}-\frac{1}{6}e^{2\varphi}-e^{\varphi-2g}\\
e^{-f}\dot{g} &= -\frac{1}{3}e^{-\varphi}-\frac{1}{6}e^{2\varphi}+e^{\varphi-2g}\\
e^{-f}\dot{\varphi} &= -\frac{1}{3}e^{-\varphi}+\frac{1}{3}e^{2\varphi}-e^{\varphi-2g}
\end{aligned}\tag{3.3}$$

In [24] these equations were partially integrated. Furthermore, it was shown numerically that there are solutions that interpolate from a region where the $D = 5$ metric is

$$ds_5^2 = \frac{1}{m^2r^2}[-dt^2+ds^2(H_3)+dr^2]\tag{3.4}$$

down to an exact $AdS_2 \times H_3$ solution given by

$$\begin{aligned}
ds_5^2 &= \frac{1}{4m^2 2^{2/3}}\left[ds^2(AdS_2)+4ds^2(H_3)\right]\\
X &= 4^{-1/3}\\
A^i &= \frac{1}{2\sqrt{2}m}\epsilon_{ijk}\omega^{jk}
\end{aligned}\tag{3.5}$$

and $B = 0$, $C = 0$. Such solutions are sometimes called topological black holes.

In [24] these solutions were uplifted on an S^5 using the results of [14] to obtain solutions of type IIB supergravity.[3] The type IIB solution obtained by uplifting the $AdS_2 \times H_3$

[3]The correctly uplifted formula, for the $AdS_2 \times H_3$ solution, were given in [25].

solution (3.5) is dual to the SCFT living on D3-branes wrapping an associative H_3 in a G_2 manifold. This CFT preserves 2 supercharges. A key aspect of this interpretation is that the gauge fields that are switched on are dual to the R-symmetry currents that must be activated in order that the field theory on the D3-branes, i.e. $N = 4$ $d = 4$ SYM theory on $\mathbb{R} \times H_3$, can preserve supersymmetry [20]. Additional evidence for this interpretation is provided by the uplifted numerical solution of [24]. It describes a "RG flow across dimensions" from the locally asymptotic AdS_5 region (3.4), where one clearly sees the $\mathbb{R} \times H_3$ world-volume of the D3-brane, down to the AdS_2 fixed point (3.5). Exactly the same kind of arguments [20] lead to the interpretation of the $AdS_5 \times_w \mathcal{N}_6$ solution with $N = 2$ supersymmetry that we mentioned at the beginning of this section.

After setting $m = 1/2$ we can use (3.1) to uplift the $AdS_2 \times H_3$ solution (3.5) and also the numerical solution found in [24] to obtain solutions of $D = 11$ supergravity. We find that the $AdS_2 \times H_3$ solution (3.5) uplifts to a $D = 11$ solution with metric given by

$$
\begin{aligned}
ds^2_{11} = \frac{(1+\sin^2\theta)^{1/3}}{2^{4/3}} \Big[& ds^2(AdS_2) + 4ds^2(H_3) + 2ds^2(H_2) \\
& +2d\theta^2 + \frac{2\sin^2\theta}{(1+\sin^2\theta)}(dx_3 + V)^2 + \frac{4\cos^2\theta}{(1+\sin^2\theta)} D\mu^i D\mu^i \Big] \qquad (3.6)
\end{aligned}
$$

where $D\mu^i = d\mu^i + \omega^{ij}\mu^j$. The numerical solution of [24] uplifts to a $D = 11$ solution that interpolates from a region with a locally AdS_5 factor as in (3.4), to the above AdS_2 solution. The dual interpretation is therefore clear. The numerical solution describes the RG flow across dimensions from the $d = 4$ CFT that lives on M5-branes wrapping a holomorphic H_2 in a CY_2, placed on $\mathbb{R} \times H_3$ with suitable R symmetry currents activated in order to preserve supersymmetry, down to a conformal quantum mechanics with two supercharges.

One might wonder what happens if one considers the CFT living on M5-branes placed directly on $\mathbb{R} \times H_3 \times H_2$. With the above R-symmetry currents, this corresponds to $M5$-branes wrapping $H_3 \times H_2$ in a $CY_3 \times CY_2$ where H_3 is a SLAG 3-cycle and H_2 is a holomorphic 2-cycle. In fact solutions describing such wrappings were already constructed in [26]. In particular, there is an AdS_2 solution which is exactly the same as the uplifted solution (3.6). Furthermore, [26] also studied some flow equations: if we substitute $e^{2f} = 2^{2/3}e^{-4\lambda/3}e^{2\bar{f}}$, $e^{2g} = 2^{5/3}e^{-4\lambda/3}e^{2g_1}$ and $e^{-\varphi} = 2^{-1/3}e^{-10\lambda/3}$ into (3.3) then we obtain the odes in equation (4.5) of [26] provided that we set $g_2 = -\lambda$.

We can also consider reversing the order. We could first consider the $d = 3$ CFT living on M5-branes wrapping a SLAG H_3 in a CY_3. The relevant $AdS_4 \times_w \mathcal{N}_7$ solution was constructed in [27]. We now consider placing this $d = 4$ CFT on $\mathbb{R} \times H_2$ with suitable R-symmetry currents to preserve supersymmetry and ask what happens in the IR. It was shown in [1] how one can carry out an explicit KK reduction from $D = 11$ supergravity on $\mathcal{N}_7$ to $D = 4$ minimal gauged supergravity. In particular, one finds that the $AdS_2 \times H_2$ solution of [28] also leads to the solution (3.6). Furthermore, the more general explicit topological black hole solution [28] uplifted on $\mathcal{N}_7$ describes the flow from the three-dimensional CFT on $\mathbb{R} \times H_2$ to the conformal quantum mechanics.

3.2 Uplifting a Maldacena-Núñez solution

Let us now consider another class of wrapped brane solutions. In [20] Maldacena and Núñez constructed supersymmetric solutions of $D = 5$ $\mathrm{U}(1)^3$ gauged supergravity which, upon lifting on an S^5 to type IIB supergravity, describe the $(2,2)$ SCFT arising on $D3$-branes wrapping a holomorphic H_2 in a Calabi-Yau three-fold. In the $D = 5$ solutions two of the three gauge-fields are equal and one of the two scalar fields vanish, which means [11] that these solutions can be recast as solutions of Romans' $D = 5$ $\mathrm{SU}(2) \times \mathrm{U}(1)$ gauge theory, with vanishing abelian gauge-field and with the non-abelian gauge-fields lying in an abelian subgroup. In our conventions, the non-trivial fields are given by

$$\begin{aligned} ds^2_5 &= \frac{e^{2f}}{m^2}[ds^2(\mathbb{R}^{1,1}) + dr^2] + \frac{e^{2g}}{m^2} ds^2(H_2) \\ X &= e^{-\varphi} \\ F^3 &= -\frac{1}{\sqrt{2}m} vol(H_2) \end{aligned} \tag{3.7}$$

and f, g, φ are functions of r that satisfy the differential equations

$$\begin{aligned} e^{-f}\dot{f} &= -\frac{2}{3}e^{-\varphi} - \frac{1}{3}e^{2\varphi} - \frac{1}{6}e^{\varphi-2g} \\ e^{-f}\dot{g} &= -\frac{2}{3}e^{-\varphi} - \frac{1}{3}e^{2\varphi} + \frac{1}{3}e^{\varphi-2g} \\ e^{-f}\dot{\varphi} &= -\frac{2}{3}e^{-\varphi} + \frac{2}{3}e^{2\varphi} - \frac{1}{6}e^{\varphi-2g} \end{aligned} \tag{3.8}$$

These equations were partially integrated in equation (17) of [20]. In [20] it was shown that there is a solution describing a flow across dimension from a locally AdS_5 region

$$ds^2_5 = \frac{1}{m^2 r^2}\left[ds^2(\mathbb{R}^{1,1}) + ds^2(H^2) + dr^2\right] \tag{3.9}$$

down to an exact $AdS_3 \times H_2$ solution given by

$$\begin{aligned} ds^2_5 &= \frac{1}{m^2 2^{4/3}}\left[ds^2(AdS_3) + ds^2(H_2)\right] \\ X &= 2^{-1/3} \\ F^3 &= -\frac{1}{\sqrt{2}m} vol(H_2) \end{aligned} \tag{3.10}$$

We can now uplift this $AdS_3 \times H_2$ solution to $D = 11$ using (3.1). The solution we obtain is particularly simple because $\bar{\Delta} = 1$ and it is precisely the solution first found in [26] which describes an M5-brane wrapping an $H_2 \times H_2$ embedded in a product of two Calabi-Yau two-folds. Moreover, the flow across dimension solution uplifts to a subclass of flow solutions studied in [26]. In particular if one sets $e^{-\varphi} = 2^{-1/3}e^{-5\lambda_2/3}$, $e^{2f} = 2^{-4/3}e^{-2\lambda_2/3}e^{2\bar{f}}$, $e^{-2g} = 2^{4/3}e^{2\lambda_2/3}e^{-2g_2}$ and substitutes into the differential equations given in (3.8), one obtains the differential equations in (3.5) of [26] after restricting to the case that $e^{2\lambda_1} = e^{-3\lambda_2}$ and $e^{2g_1} = e^{-\lambda_2}$.

3.3 Uplifting the Klemm-Sabra magnetic string solution

As we have already noted, any solution of minimal $D = 5$ gauged supergravity is also a solution of Romans' $D = 5$ theory. Consider the supersymmetric magnetic string solution of [29], which we can write

$$
\begin{aligned}
ds^2 &= r^{1/2}\left(\frac{r}{3}-\frac{1}{r}\right)^{3/2} ds^2(\mathbb{R}^{1,1}) + \frac{1}{9m^2}\left(\frac{r}{3}-\frac{1}{r}\right)^{-2} dr^2 + \frac{1}{9m^2} r^2 ds^2(H_2) \\
G &= \frac{1}{\sqrt{2}}F^3 = -\frac{1}{3m} vol(H_2) \\
X &= 1
\end{aligned}
\tag{3.11}
$$

This interpolates from an asymptotic locally AdS_5 region, with spatial slices $\mathbb{R}^{1,1} \times H_2$ in Poincaré coordinates, to an $AdS_3 \times H_2$ solution, which can be written

$$
\begin{aligned}
ds^2 &= \frac{4}{9m^2}\left[ds^2(AdS_3) + \frac{3}{4} ds^2(H_2)\right] \\
G &= \frac{1}{\sqrt{2}}F^3 = -\frac{1}{3m} vol(H_2) \\
X &= 1
\end{aligned}
\tag{3.12}
$$

These solutions can be uplifted on[4] an S^5 to type IIB supergravity using the formulae in [11]. The uplifted solutions are dual to D3-branes wrapping a holomorphic H_2 embedded in a CY_4 (see [31]). In particular the uplifted solution (3.11) describes the flow across dimension from the AdS_5 region down to the AdS_3 fixed point, which is dual to a $(0, 2)$ SCFT.

The solution (3.11) can also be uplifted to $D = 11$ using (3.1). It then describes a flow across dimension of the $d = 4$ SCFT living on M5-branes wrapped on H_2 in CY_2, placed on $\mathbb{R}^{1,1} \times H_2$, down to a $d = 2$ $(0, 2)$ SCFT. As far as we can tell this is a new solution of $D = 11$ supergravity.

We also note, as somewhat of an aside, that the Klemm-Sabra solution can also be uplifted to $D = 11$ in another way. First recall the $N = 1$ $AdS_5 \times_w \mathcal{N}_6$ solution of $D = 11$ supergravity which describes the $N = 1$ $d = 4$ CFT arising on M5-branes wrapping a holomorphic H_2 in a CY_3 [20]. The consistent KK reduction on this $\mathcal{N}_6$ down to minimal $D = 5$ gauged supergravity was carried out in [6]. The solution (3.11), thought of as a solution of minimal gauged supergravity, can thus be uplifted on $\mathcal{N}_6$. The resulting $D = 11$ solution describes the flow across dimension of the $d = 4$ CFT placed on $\mathbb{R}^{1,1} \times H_2$ down to the $d = 2$ (0,2) CFT which is dual to the uplifted $AdS_3 \times H_2$ solution (3.12). Moreover, one finds that the uplifted flow solution and the uplifted AdS_3 solution are identical to the corresponding $D = 11$ solutions that describe M5-branes wrapping a holomorphic $H_2 \times H_2$ in a CY_4 which were found in [27].

This story can be generalised further by noting that the solution of [20] is just one example of several infinite classes of explicit $AdS_5 \times \mathcal{N}_6$ solutions of $D = 11$ supergravity

[4] Using the results of [3] we can uplift on an arbitrary five-dimensional Sasaki-Einstein space; the resulting AdS_3 solutions have already been presented in [30].

that were found in [7], all of which are dual to $N = 1$ $d = 4$ CFTs. The results of [6] allow us to uplift the Klemm-Sabra solution [29] on any of these $\mathcal{N}_6$. The resulting $D = 11$ solutions are dual to the flow across dimension of the $d = 4$ CFTs on $\mathbb{R}^{1,1} \times H_2$ down to $d = 2$ (0,2) CFTs which are dual to the uplifted $AdS_3 \times H_2$ solutions. In particular, if one uplifts the $AdS_3 \times H_2$ solution on these $\mathcal{N}_6$ one finds solutions that should be included in the general constructions of [30], but we have not checked this in detail.

3.4 Uplifting Romans' magnetovac solutions

The solutions of Romans' theory that we discussed in the previous two subsections are in fact special cases of a more general class of supersymmetric magnetovac solutions on $AdS_3 \times S^2$, T^2 and H_2 that were first constructed (earlier) by Romans in [16]. The non-abelian SU(2) gauge fields lie in an abelian subgroup and in addition the U(1) gauge field is, in general, also active. Specifically, the solutions, which are parametrised by the positive constant x, can be written

$$\begin{aligned} ds_5^2 &= \frac{4x^{4/3}}{m^2(2x+1)^2}\left[ds^2(AdS_3) + R_{(2)}^2 ds^2(\Sigma_l)\right] \\ G &= -\frac{4(2x-1)}{m(2x+1)^2} R_{(2)}^2 vol(\Sigma_l) \\ F^3 &= -\frac{8x}{\sqrt{2}m(2x+1)^2} R_{(2)}^2 vol(\Sigma_l) \\ X &= x^{1/3} \end{aligned} \tag{3.13}$$

with $l = 0, \pm 1$ and Σ_l is T^2, S^2, H^2, respectively. This is a supersymmetric solution provided that

$$l = \frac{4(1-4x)}{(2x+1)^2} R_{(2)}^2 \tag{3.14}$$

In particular, when $0 < x < 1/4$ we take $l = 1$, when $x = 1/4$ we take $l = 0$ and when $x > 1/4$ we take $l = -1$. When $x = 1/4$ we can set $R_{(2)}^2 = 1$ after scaling the T^2.

Note that when $x = 1/2$ the U(1) gauge field vanishes and the corresponding $AdS_3 \times H_2$ solution is precisely the $AdS_3 \times H_2$ solution (3.10) that we discussed above. This solution actually preserves twice as much supersymmetry as the generic solution. On the other hand when $x = 1$ we get the $AdS_3 \times H_2$ solution of minimal gauged supergravity that we presented in (3.12).

The general magnetovacs can all be uplifted to $D = 11$ using (3.1) to obtain new supersymmetric solutions of $D = 11$ supergravity. It would be interesting to study these solutions further. It would be interesting to see if the solutions lie in the class of AdS_3 solutions that arise from the "Kähler-4" class of Minkowski solutions that were discussed in [32].

The general magnetovac solutions (3.13) can also be uplifted to type IIB on an S^5 using the results of [14]. We present the explicit results in the appendix where we also carry out an independent check of the preservation of supersymmetry using the results of [33].

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4. Final comments

Through inspired guesswork we have constructed the non-linear KK ansatz, at the level of bosonic fields, for the reduction of $D = 11$ supergravity to Romans' $D = 5$ gauged supergravity using the most general $AdS_5 \times_w \mathcal{N}_6$ solutions of $D = 11$ supergravity that are dual to $N = 2$ SCFTs in $d = 4$. Invoking the argument of [34] we can conclude that it should be possible to extend this result at the level of the fermions, though doing this explicitly would be very difficult. A less ambitious goal would be to show that for the bosonic configurations that we are considering, a supersymmetric solution of the Romans' theory uplifts to a supersymmetric solution of $D = 11$ supergravity. This type of result was shown for other cases in [3, 6, 1].

Another extension of this work would be to show that for the most general $AdS_5 \times_w \mathcal{N}_5$ solutions of type IIB supergravity that are dual to $N = 2$ $d = 4$ SCFTs, there is also a consistent KK reduction to Romans' theory. However, before this can be investigated, using the techniques of this paper, the classification of such solutions, refining the results of [5], needs to be carried out.

There is now substantial evidence that the conjecture of [1] concerning consistency of KK truncations is correct, having been verified in several cases. It would be nice to have a rigorous supergravity proof (perhaps building on the work of [35]) independent of a case by case construction. Ideally, such an analysis would provide an algorithmic prescription for constructing the non-linear KK ansatz, which, so far, has been found essentially by trial and error. It would also be nice to have a general proof from the dual SCFT point of view and some discussion in this direction has appeared in [36].

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A. Consistency of the KK ansatz

In this appendix we provide some details of the proof that the KK reduction ansatz (2.17), (2.22) is indeed consistent, i.e., that it satisfies the equations of motion of $D = 11$ supergravity, provided that the field equations of Romans' $D = 5$ $\mathrm{SU}(2) \times \mathrm{U}(1)$ gauged supergravity are imposed.

Our conventions for $D = 11$ supergravity follow those of [22]. In particular, the equations of motion are given by

$$\begin{aligned} \mathrm{d}G_4 &= 0\,, && \text{(A.1)}\\ \mathrm{d}*_{11} G_4 &= -\frac{1}{2} G_4 \wedge G_4\,, && \text{(A.2)}\\ R_{AB} &= T_{AB}\,, && \text{(A.3)} \end{aligned}$$

where we have defined

$$T_{AB} = \frac{1}{12} G_{4AC_1C_2C_3} G_{4B}{}^{C_1C_2C_3} - \frac{1}{144} g_{AB} G_{4C_1C_2C_3C_4} G_4^{C_1C_2C_3C_4}\,. \tag{A.4}$$

The frame of the deformed metric (2.17) is taken to be

$$\begin{aligned} \bar{e}^\mu &= \lambda^{-1/2} X^{-1/6} \Delta^{1/6} e^\mu\\ \bar{e}^1 &= X^{1/3} \Delta^{1/6} e^1\\ \bar{e}^2 &= X^{1/3} \Delta^{1/6} e^2\\ \bar{e}^3 &= X^{5/6} \Delta^{-1/3} \hat{e}^3\\ \bar{e}^4 &= X^{1/3} \Delta^{1/6} e^4\\ \bar{e}^a &= X^{-2/3} \Delta^{-1/3} \frac{\lambda\rho}{2m} (f^a + \sqrt{2} m k_i^a A^i) \end{aligned} \tag{A.5}$$

where e^μ, $\mu = 0, \ldots, 4$, is a frame for the five-dimensional metric $\mathrm{d}s_5^2$, $(e^1, \ldots, e^6)$ is the orthonormal frame for the internal undeformed space $\mathcal{N}_6$ introduced in subsection 2.1, $\hat{e}^3$ is given in (2.20), f^a, $a = 5, 6$, is a frame for the unit two sphere and k_i^a are the components of the three Killing vectors on this two-sphere with respect to the frame f^a. These Killing vectors satisfy the SU(2) Lie algebra

$$[k_i, k_j] = \epsilon_{ijk} k_k \tag{A.6}$$

and also

$$k^{ia} k_a^j = \delta^{ij} - \mu^i \mu^j\,, \qquad k_i^a k_i^b = \delta^{ab}\,. \tag{A.7}$$

It is also useful to rewrite $D\mu^i$ in (2.20) as

$$D\mu^i = \epsilon^{ab} k_b^i (f_a + \sqrt{2} m k_a^k A^k) \tag{A.8}$$

and to note that

$$DD\mu^i = \sqrt{2} m \epsilon_{ijk} F^k \mu^j = \sqrt{2} m \epsilon^{ab} k_b^i k_a^k F^k\,. \tag{A.9}$$

A.1 The four-form equations

The KK ansatz (2.22) for the four-form satisfies the Bianchi identity (A.1) provided the $D = 5$ gauge fields satisfy the Bianchi identities that can be obtained by differentiating (2.8), and that the field equation (2.11) for the SU(2) field strength is imposed. In order to verify this one needs to use the relations (2.5) and that the differentials of $\hat{e}^3$ and $D\mu^i$ give field strength contributions, as in (A.9).

To check that the KK ansatz (2.17), (2.22) satisfies the $D = 11$ four-form equation of motion (A.2) is somewhat more involved. Imposing the $D = 5$ field equation (2.12) for simplicity, the Hodge dual, with respect to the deformed metric (2.17), of (2.22) reads

$$\begin{aligned}
*_{11}\, G_4 = {} & *_{11}\tilde{G}_4 \; + X^4 * G \wedge \lambda^{1/2}\rho\, \hat{e}^{1234} \\
& + \frac{1}{2\sqrt{2}m} X^{-2} * F^i \wedge \Big[\rho\sqrt{1-z}\epsilon_{ijk}\mu^j D\mu^k \wedge e^{124} \\
& \qquad - \frac{1}{4m} z\mu_i \epsilon_{hjk}\mu^h D\mu^j \wedge D\mu^k \wedge [\lambda^{-2}e^{12} + \lambda^{-1/2}\rho X \Delta^{-1}\hat{e}^{34}]\Big] \\
& + \frac{1}{2\sqrt{2}m} F^i \wedge \rho\, \hat{e}^{123} \wedge \Big[\lambda^{1/2}\; \epsilon_{ijk}\mu^j D\mu^k \wedge e^4 \\
& \qquad - \frac{1}{4m} X^{-2}\Delta^{-1}\sqrt{z(1-z)}\; \mu_i \epsilon_{hjk}\mu^h D\mu^j \wedge D\mu^k\Big] \\
& + \Big[\frac{i}{16\sqrt{2}m^3} F \wedge z\; \epsilon_{ijk}\mu^i D\mu^j \wedge D\mu^k \wedge (e^1 - ie^2) \wedge (\lambda^{-2}e^4 + iX\Delta^{-1}\lambda^{-1/2}\rho\hat{e}^3) \\
& - \frac{1}{16\sqrt{2}m^2} C \wedge X^{-2}\Delta^{-1}\rho\sqrt{z(1-z)}\; \epsilon_{ijk}\mu^i D\mu^j \wedge D\mu^k \wedge (e^1 - ie^2) \wedge \hat{e}^{34} + c.c.\Big]\,,
\end{aligned} \tag{A.10}$$

where $*_{11}\tilde{G}_4$ is the Hodge dual, with respect to the metric (2.17), of (2.23), namely,

$$\begin{aligned}
*_{11}\, \tilde{G}_4 = {} & 3X^{-1} * \mathrm{d}X \wedge \rho\sqrt{1-z}e^{124} \\
& \frac{1}{\lambda^{9/2}\rho^2} * \mathbb{1} \wedge \Big\{ \frac{3\lambda\rho^2}{\sqrt{1-z}} X^{-1} *_6 \big[[(\mathrm{d}\lambda)_4 e^{12} - (\mathrm{d}\lambda)_2 e^{14} + (\mathrm{d}\lambda)_1 e^{24}] \wedge e^{56}\big] \\
& + X^{-2}\frac{\lambda^{1/2}}{\sqrt{1-z}} *_6 \big[[\Delta \mathrm{d}(z\rho) + \rho(1-z)(X - X^{-2})\mathrm{d}z] \wedge e^{356}\big] \\
& - 2m(X - X^{-2})z\big[\lambda\rho X e^{34} + \lambda^{-1/2}\Delta e^{12}\big]\Big\}\,.
\end{aligned} \tag{A.11}$$

Here, $*_6$ is the Hodge dual with respect to the undeformed metric $ds^2(\mathcal{N}_6)$ in (2.1). In fact, the presence in (A.11) of the volume form $*\mathbb{1}$, corresponding to the spacetime metric $\mathrm{d}s_5^2$, allows one to write (A.11) in terms of the frame $(e^1, \ldots, e^6)$ of the undeformed metric $ds^2(\mathcal{N}_6)$, once the contributions from the scalar field X have been taken into account.

Computing the exterior derivative of (A.11) we find

$$\mathrm{d} *_{11} \tilde{G}_4 = \big[3\mathrm{d}(X^{-1} * \mathrm{d}X) + 4m^2(X^2 - X^{-1}) * \mathbb{1}\big] \wedge \rho\sqrt{1-z}\; e^{124}\,, \tag{A.12}$$

where we used the field equation for the undeformed four-form (2.4), which can be written

$$\begin{aligned}
\mathrm{d}\Big\{ & \frac{1}{\lambda^{9/2}\rho^2\sqrt{1-z}} \Big[\lambda^{1/2} *_6 [\mathrm{d}(z\rho) \wedge e^{356}] \\
& + 3\lambda\rho^2 *_6 \big[[(\mathrm{d}\lambda)_4 e^{12} - (\mathrm{d}\lambda)_2 e^{14} + (\mathrm{d}\lambda)_1 e^{24}] \wedge e^{56}\big]\Big]\Big\} = 0\,.
\end{aligned} \tag{A.13}$$

Next, differentiating (A.10) with the help of (A.12) and (2.5), and wedging (2.22) with itself, one can compute

$$
\begin{aligned}
\mathrm{d}*_{11} G_4+\frac{1}{2}G_4\wedge G_4 = &\Big[3\mathrm{d}(X^{-1}*\mathrm{d}X)-X^4*G\wedge G+\frac{1}{2}X^{-2}(*F^i\wedge F^i+*C\wedge\bar{C})\\
&\qquad +4m^2(X^2-X^{-1})*1\!\!1\Big]\wedge\rho\sqrt{1-z}e^{124}\\
&+\Big[d(X^4*G)+\frac{1}{2}F^i\wedge F^i+\frac{1}{2}\bar{C}\wedge C\Big]\wedge\lambda^{1/2}\rho\,\hat{e}^{1234}\\
&+\frac{1}{2\sqrt{2}m}\Big[D(X^{-2}*F^i)+F^i\wedge G\Big]\wedge\Big[\rho\sqrt{1-z}\epsilon_{ijk}\mu^j D\mu^k\wedge e^{124}\\
&\qquad -\frac{1}{4m}z\mu_i\epsilon_{hjk}\mu^h D\mu^j\wedge D\mu^k\wedge[\lambda^{-2}e^{12}+\lambda^{-1/2}\rho X\Delta^{-1}\hat{e}^{34}]\Big].
\end{aligned}
\tag{A.14}
$$

This indeed shows that the KK ansatz (2.17), (2.22) satisfies the $D = 11$ four-form equation of motion (A.2), provided that the five-dimensional fields satisfy the field equations (2.9)–(2.12) of Romans' $D = 5$ $\mathrm{SU}(2)\times\mathrm{U}(1)$ gauged supergravity.

A.2 The Einstein equations

In order to check the $D = 11$ Einstein equations (A.3), we first give explicit expressions for the tensor T_{AB}, that defines the right hand side, in terms of the frame (A.5). Substituting the expression (2.22) of the KK ansatz for G_4 into (A.4) we find, for the external components,

$$
\begin{aligned}
\lambda^{-1}X^{-1/3}\Delta^{1/3}T_{\mu\nu}=&\frac{zX^5}{2\Delta}\Big(G_{\mu\rho}G_\nu{}^\rho-\frac{1}{6}\eta_{\mu\nu}G_{\rho\sigma}G^{\rho\sigma}\Big)-\frac{1}{24}((1-z)X^{-4}\Delta^{-1}+2X^{-2})\eta_{\mu\nu}F^i_{\rho\sigma}F^{i\rho\sigma}\\
&+\frac{1}{4}(X^{-2}+(1-z)X^{-4}\Delta^{-1})F^i_{\mu\rho}F^{i\rho}_\nu+\frac{1}{4}X^{-1}z\Delta^{-1}F^i_{\mu\rho}F^{j\rho}_\nu\mu^i\mu^j\\
&\frac{1}{2}X^{-2}\Big[C_{(\mu}{}^\rho\bar{C}_{\nu)\rho}-\frac{1}{6}\eta_{\mu\nu}C_{\rho\sigma}\bar{C}^{\rho\sigma}\Big]-\frac{1}{24}(1-z)X^{-4}\Delta^{-1}\eta_{\mu\nu}C_{\rho\sigma}\bar{C}^{\rho\sigma}\\
&+\frac{3}{2}(1-z)z\Delta^{-2}X^{-3}[3\nabla_\mu X\nabla_\nu X-\eta_{\mu\nu}(\nabla X)^2]-\frac{3}{2}\frac{\Delta+zX}{\Delta^2X^3(1-z)\lambda^3}(\nabla\lambda)^2\\
&-\frac{2m\rho(X(2+z)+X^{-2}(1-z))}{\sqrt{1-z}\Delta^2X^3}(d\lambda)_4\\
&-\frac{2m^2}{3\Delta^2X^5}[2z^2X^9+z(7-5z)X^6+(9-8z+4z^2)X^3+z(1-z)]
\end{aligned}
\tag{A.15}
$$

where we note, for example, that $G_{\mu\nu}$ are the components of G with respect to the $D = 5$ frame e^μ. For the mixed components we find

$$
\begin{aligned}
T_{\mu1} &= \frac{9}{2}X^{-13/6}\Delta^{-7/3}\lambda^{-1/2}z\ (\mathrm{d}\lambda)_1\ \nabla_\mu X\ ,\\
T_{\mu2} &= \frac{9}{2}X^{-13/6}\Delta^{-7/3}\lambda^{-1/2}z\ (\mathrm{d}\lambda)_2\ \nabla_\mu X\ ,\\
T_{\mu3} &= X^{4/3}\Delta^{-5/6}\lambda\sqrt{1-z}\Big[-\frac{3}{2}z\Delta^{-1}\ G_{\mu\nu}\nabla^\nu X+\frac{1}{16}X^{-4}\epsilon_{\mu\lambda\nu\rho\sigma}(F^{i\lambda\nu}F^{i\rho\sigma}+C^{\lambda\nu}\bar{C}^{\rho\sigma})\Big],
\end{aligned}
$$

$$
\begin{aligned}
T_{\mu 4} &= X^{-7/6}\Delta^{-7/3}\lambda^{-1/2}z\left[\frac{9}{2}X^{-1}(\mathrm{d}\lambda)_4 + 3m\rho^{-1}\sqrt{1-z}\,\left[X^2 z + X^{-1}(3-z)\right]\right]\nabla_\mu X,\\
T_{\mu a} &= \frac{1}{2\sqrt{2}}X^{-1/6}\Delta^{-5/6}\lambda^{5/2}\rho\; k_a^i\big[3(1-z)X^{-3}\Delta^{-1}\;F^i_{\mu\nu}\nabla^\nu X + \frac{1}{4}X^2\epsilon_{\mu\lambda\nu\rho\sigma}F^{\lambda\nu}G^{\rho\sigma}\big]
\end{aligned}
\tag{A.16}
$$

Finally, to write the internal components of T_{AB} it proves convenient to introduce, for $n = 1, 2, 4$,

$$
\begin{aligned}
U_n &= \frac{\lambda^{1/2}}{\sqrt{1-z}}X^{1/6}\Delta^{-7/6}\big[3\rho X^{-1}(\mathrm{d}\lambda)_n + 2m\sqrt{1-z}X[Xz + (3-z)X^{-2}]\delta_{n4}\big]\;,\\
V_n &= \frac{1}{\lambda\sqrt{1-z}}X^{2/3}\Delta^{-2/3}\big[3X^{-2}(\mathrm{d}\lambda)_n - 2m\lambda^3\rho\sqrt{1-z}(X - X^{-2})\;\delta_{n4}\big]\;,
\end{aligned}
\tag{A.17}
$$

so that (2.23) can be written in the frame (A.5) as

$$
\tilde{G}_4 = (V_4\bar{e}^{12} + U_1\bar{e}^{13} - V_2\bar{e}^{14} + U_2\bar{e}^{23} + V_1\bar{e}^{24} - U_4\bar{e}^{34})\wedge\bar{e}^{56}\,. \tag{A.18}
$$

Then one finds, for the non-vanishing internal components ($m, n, p \in \{1, 2, 4\}$, $a, b \in \{5, 6\}$):

$$
\begin{aligned}
T_{mn} &= \frac{1}{12}X^{4/3}\Delta^{-4/3}\lambda z\big[-X^4G_{\mu\nu}G^{\mu\nu} + \frac{1}{2}X^{-2}\left(F^i_{\mu\nu}F^{i\mu\nu} + C_{\mu\nu}\bar{C}^{\mu\nu}\right)\big]\;\delta_{mn}\\
&\quad -\frac{3}{2}\lambda z(1-z)\Delta^{-7/3}X^{-8/3}\nabla_\mu X\nabla^\mu X\;\delta_{mn}\\
&\quad +\frac{1}{2}(U_mU_n - V_mV_n) + \frac{1}{6}(-U_pU^p + 2V_pV^p)\;\delta_{mn},\\
T_{13} &= \frac{1}{2}(U_4V_2 - U_2V_4),\\
T_{23} &= \frac{1}{2}(U_1V_4 - U_4V_1),\\
T_{33} &= -\frac{1}{12}\lambda zX^{16/3}\Delta^{-4/3}G_{\mu\nu}G^{\mu\nu}\\
&\quad +\frac{1}{24}\lambda X^{-5/3}\Delta^{-4/3}[Xz + 3X^{-2}(1-z)]\left(F^i_{\mu\nu}F^{i\mu\nu} + C_{\mu\nu}\bar{C}^{\mu\nu}\right)\\
&\quad +3\lambda z(1-z)\Delta^{-7/3}X^{-8/3}\nabla_\mu X\nabla^\mu X\\
&\quad +\frac{1}{6}(-U_pU^p + 2V_pV^p)\;,\\
T_{3a} &= \frac{1}{4\sqrt{2}}X^{5/6}\Delta^{-4/3}\lambda^{5/2}\rho\sqrt{1-z}\;k^i_aF^i_{\mu\nu}G^{\mu\nu}\;,\\
T_{ab} &= -\frac{1}{6}\lambda zX^{4/3}\Delta^{-4/3}\left[-X^4G_{\mu\nu}G^{\mu\nu} + \frac{1}{2}X^{-2}\left(F^i_{\mu\nu}F^{i\mu\nu} + C_{\mu\nu}\bar{C}^{\mu\nu}\right)\right]\delta_{ab}\\
&\quad +\frac{1}{8}X^{-2/3}\Delta^{-4/3}\lambda zk^i_ak^j_bF^i_{\mu\nu}F^{j\mu\nu}\\
&\quad +3\lambda z(1-z)\Delta^{-7/3}X^{-8/3}\nabla_\mu X\nabla^\mu X\delta_{ab}\\
&\quad +\frac{1}{3}(-U_pU^p + 2V_pV^p)\delta_{ab}\,.
\end{aligned}
\tag{A.19}
$$

The spin connection corresponding to the deformed metric (2.17) can be computed in the frame (A.5) and we find, for the external components,

$$\begin{aligned}\bar\omega^{\mu\nu} &= \omega^{\mu\nu} + \lambda^{1/2}X^{-17/6}\Delta^{-7/6}(1-z)\nabla^{[\mu}X\ \bar e^{\nu]} - \frac{1}{2}X^{7/6}\Delta^{-2/3}\lambda^{1/2}\sqrt{1-z}G^{\mu\nu}\bar e^3 \\ &\quad -\frac{1}{2\sqrt2}X^{-1/3}\Delta^{-2/3}\lambda^2\rho\ k^i_a F^{i\mu\nu}\bar e^a\ ,\end{aligned} \tag{A.20}$$

for the mixed components,

$$\begin{aligned}\bar\omega^{\mu1} &= -\frac{1}{2}X^{-7/3}\Delta^{-7/6}\lambda^{-1}(\mathrm{d}\lambda)_1\ \bar e^\mu\ -\frac12\lambda^{1/2}zX^{1/6}\Delta^{-7/6}\nabla^\mu X\ \bar e^1\\ \bar\omega^{\mu2} &= -\frac{1}{2}X^{-7/3}\Delta^{-7/6}\lambda^{-1}(\mathrm{d}\lambda)_2\ \bar e^\mu\ -\frac12\lambda^{1/2}zX^{1/6}\Delta^{-7/6}\nabla^\mu X\ \bar e^2\\ \bar\omega^{\mu3} &= -\frac12X^{7/6}\Delta^{-2/3}\lambda^{1/2}\sqrt{1-z}\ G^\mu{}_\nu\ \bar e^\nu\\ &\qquad -\frac12\lambda^{1/2}X^{-5/6}\Delta^{-7/6}[Xz+3X^{-2}(1-z)]\nabla^\mu X\ \bar e^3\\ \bar\omega^{\mu4} &= X^{-1/3}\Delta^{-7/6}\lambda^{-1}\left[-\frac12X^{-2}(\mathrm{d}\lambda)_4+\frac{2mz}{3\rho}\sqrt{1-z}(X-X^{-2})\right]\bar e^\mu\\ &\qquad\qquad -\frac12\lambda^{1/2}zX^{1/6}\Delta^{-7/6}\nabla^\mu X\ \bar e^4\\ \bar\omega^{\mu a} &= -\frac{1}{2\sqrt2}X^{-1/3}\Delta^{-2/3}\lambda^2\rho k^{ai}F^{i\mu}{}_\nu\ \bar e^\nu\ +\lambda^{1/2}zX^{1/6}\Delta^{-7/6}\nabla^\mu X\ \bar e^a\end{aligned} \tag{A.21}$$

and for the non-vanishing internal components,

$$\begin{aligned}\bar\omega^{12} &= -mB+M_2\bar e^1-M_1\bar e^2-N_4\bar e^3\\ \bar\omega^{13} &= P_4\bar e^2-Q_1\bar e^3-P_2\bar e^4\\ \bar\omega^{14} &= M_4\bar e^1+N_2\bar e^3-M_1\bar e^4\\ \bar\omega^{1a} &= -R_1\bar e^a\\ \bar\omega^{23} &= -P_4\bar e^1-Q_2\bar e^3+P_1\bar e^4\\ \bar\omega^{24} &= M_4\bar e^2-N_1\bar e^3-M_2\bar e^4\\ \bar\omega^{2a} &= -R_2\bar e^a\\ \bar\omega^{34} &= -P_2\bar e^1+P_1\bar e^2+Q_4\bar e^3\\ \bar\omega^{4a} &= -R_4\bar e^a\\ \bar\omega^{56} &= \mu^iA^i-\frac{\mu^3}{\sqrt{(\mu^1)^2+(\mu^2)^2}}f^6\end{aligned} \tag{A.22}$$

where f^a, $a=5,6$, was introduced in (A.5) and we have defined, for $n=1,2,4$:

$$\begin{aligned}M_n &= \frac{1}{6\lambda(1-z)}X^{-1/3}\Delta^{-7/6}\Big[\left[9Xz+6X^{-2}(1-z)\right](\mathrm{d}\lambda)_n\\ &\qquad\qquad +2m\lambda^3\rho\sqrt{1-z}\left[X(2+z)+X^{-2}(1-z)\right]\delta_{n4}\Big],\\ N_n &= -\frac{\lambda^{1/2}}{2(1-z)}X^{-5/6}\Delta^{-2/3}\Big[3\rho X(\mathrm{d}\lambda)_n+2m\sqrt{1-z}\left[X(1+z)+X^{-2}(1-z)\right]\delta_{n4}\Big],\end{aligned}$$

$$
\begin{aligned}
P_n &= \frac{\lambda^{1/2}}{2(1-z)} X^{1/6}\Delta^{-2/3}\Big[3\rho(\mathrm{d}\lambda)_n + 2m\sqrt{1-z}\;\delta_{n4}\Big], \\
Q_n &= -\frac{1}{6\lambda(1-z)} X^{-1/3}\Delta^{-7/6}\Big[\left[9Xz + 3X^{-2}(1-z)\right](\mathrm{d}\lambda)_n \\
&\qquad\qquad +4m\lambda^3\rho\sqrt{1-z}\left[X(2+z) + X^{-2}(1-z)\right]\delta_{n4}\Big], \\
R_n &= X^{-1/3}\Delta^{-7/6}\Big[\lambda^{-1}X^{-2}(\mathrm{d}\lambda)_n + \frac{2m}{3\lambda\rho}\sqrt{1-z}\left[Xz + X^{-2}(3-z)\right]\delta_{n4}\Big].
\end{aligned}
\tag{A.23}
$$

Notice that, when $X = 1$, $B = A^i = 0$, the spin connection reduces to that of the undeformed metric (2.1). In particular, the internal components (A.22) reduce to the spin connection of the undeformed $ds^2(\mathcal{N}_6)$, that can be calculated from the equations (2.5).

The Ricci tensor corresponding to the deformed metric (2.17) can now be calculated in the frame (A.5) and, for illustration, we just record here the expression for its external components $\bar{R}_{\mu\nu}$. To do this it is convenient to notice that, for any of the solutions $AdS_5 \times_w \mathcal{N}_6$ described in subsection 2.1, one has

$$
\nabla^2\lambda + 4m^2\lambda^2 + \frac{13z-1}{2\lambda(1-z)}(\nabla\lambda)^2 + \frac{12mz}{\lambda\rho\sqrt{1-z}}(\mathrm{d}\lambda)_4 = 0, \tag{A.24}
$$

as can be shown using (2.5). Defining the tensor

$$
\begin{aligned}
E_{\mu\nu} &= R_{\mu\nu} - 3X^{-2}\,\nabla_\mu X\,\nabla_\nu X + \frac{4}{3}m^2\left(X^2 + 2X^{-1}\right)\eta_{\mu\nu} \\
&\quad -\frac{1}{2}X^4\left[G_\mu{}^\rho G_{\nu\rho} - \frac{1}{6}\eta_{\mu\nu}\,G_{\rho\sigma}G^{\rho\sigma}\right] - \frac{1}{2}X^{-2}\left[F^i_\mu{}^\rho F^i_{\nu\rho} - \frac{1}{6}\eta_{\mu\nu}\,F^i_{\rho\sigma}F^{i\rho\sigma}\right] \\
&\quad -\frac{1}{2}X^{-2}\left[C_{(\mu}{}^\rho\,\bar{C}_{\nu)\rho} - \frac{1}{6}\eta_{\mu\nu}\,C_{\rho\sigma}\bar{C}^{\rho\sigma}\right],
\end{aligned}
\tag{A.25}
$$

and the scalar

$$
S = 3\nabla_\mu(X^{-1}\nabla^\mu X) + 4m^2(X^2 - X^{-1}) - \frac{1}{2}X^4 G_{\mu\nu}G^{\mu\nu} + \frac{1}{4}X^{-2}F^i_{\mu\nu}F^{i\mu\nu} + \frac{1}{4}X^{-2}C_{\mu\nu}\bar{C}^{\mu\nu}, \tag{A.26}
$$

a long calculation reveals that

$$
\bar{R}_{\mu\nu} = \lambda X^{1/3}\Delta^{-1/3}\left[E_{\mu\nu} + \eta_{\mu\nu}\frac{(1-z)}{6X^2\Delta}S\right] + T_{\mu\nu}, \tag{A.27}
$$

where $T_{\mu\nu}$ is given in (A.15). This shows that the external components of the $D = 11$ Einstein equations (A.3) are satisfied provided $S = 0$ and $E_{\mu\nu} = 0$, which are precisely the scalar (2.9) and Einstein equations (2.13) of Romans' $D = 5$ gauged supergravity.

B. The magnetovac solutions uplifted to type IIB

After uplifting the magnetovac solutions (3.13) to type IIB using [14] we find that the ten-dimensional metric is given by

$$
\begin{aligned}
m^2 ds^2_{10} &= \frac{4x}{(2x+1)^2}\bar{\Delta}^{1/2}\left[ds^2(AdS_3) + R^2_{(2)}ds^2(\Sigma_l)\right] + \bar{\Delta}^{1/2}d\xi^2 + \frac{\cos^2\xi}{4\bar{\Delta}^{1/2}}d\Omega_2 \\
&\quad + \frac{\cos^2\xi}{4\bar{\Delta}^{1/2}}\left[\sigma_3 - \frac{8xR^2_{(2)}}{(2x+1)^2}W\right]^2 + \frac{x\sin^2\xi}{\bar{\Delta}^{1/2}}\left[d\tau - \frac{4(2x-1)R^2_{(2)}}{(2x+1)^2}W\right]^2
\end{aligned}
\tag{B.1}
$$

where $\bar{\Delta} = \sin^2\xi + x\cos^2\xi$ and the potential W is defined so that $dW = vol(\Sigma_l)$.

We can directly check the supersymmetry of this uplifted IIB solution by recasting it in the general form found in [33]. In particular the solutions are dual to SCFTs with (at least) $(0,2)$ supersymmetry. To do this we first employ a coordinate transformation so that

$$\begin{aligned} \sigma_3 &= \sigma_3' + \frac{2x}{2x+1}dz \\ d\tau &= \frac{1}{2x+1}dz \end{aligned} \tag{B.2}$$

We then find that the metric takes the form

$$m^2 ds^2_{10} = e^{2A}\left[ds^2(AdS_3) + e^{-4A}ds^2_6 + \frac{1}{4}(dz+P)^2\right] \tag{B.3}$$

where

$$\begin{aligned} e^{2A} &= \frac{4x}{(2x+1)^2}\bar{\Delta}^{1/2} \\ e^{-4A}ds^2_6 &= R^2_{(2)}ds^2(\Sigma_l) + \frac{(2x+1)^2}{4x}d\xi^2 \\ &\quad + \frac{(2x+1)^2\cos^2\xi}{16x\bar{\Delta}}d\Omega_2 + \frac{(2x+1)^2\cos^2\xi\sin^2\xi}{16x\bar{\Delta}^2}\left[\sigma_3' + \frac{16x(x-1)R^2_{(2)}}{(2x+1)^2}W\right]^2 \\ P &= -\frac{4R^2_{(2)}}{(2x+1)\bar{\Delta}}[(2x-1)\sin^2\xi + x\cos^2\xi]W + \frac{(2x+1)\cos^2\xi}{2\bar{\Delta}}\sigma_3' \end{aligned} \tag{B.4}$$

After some calculation one can show that ds^2_6 is Kähler, with Kähler form J_6 given by

$$\begin{aligned} e^{-4A}J_6 &= R^2_{(2)}vol(\Sigma_l) + \frac{(2x+1)^2\cos^2\xi}{16x\bar{\Delta}}vol(S^2) \\ &\quad - \frac{(2x+1)^2\cos\xi\sin\xi}{8x\bar{\Delta}}d\xi\wedge\left[\sigma_3' + \frac{16x(x-1)R^2_{(2)}}{(2x+1)^2}W\right] \end{aligned} \tag{B.5}$$

and that P is a Ricci form potential for this Kähler metric. Further calculation shows that the warp factor satisfies

$$R = 8e^{-4A} \tag{B.6}$$

and that ds^2_6 satisfies

$$\nabla^2 R + R_{ij}R^{ij} - \frac{1}{2}R^2 = 0 \tag{B.7}$$

This verifies that the IIB solutions preserve $(0,2)$ supersymmetry [33].

We have already noted that for $x=1$ the solution corresponds to D3-branes wrapped on a H_2 in a CY_4 and is dual to a $(0,2)$ SCFT. The $x=1/2$ solution corresponds to D3-branes wrapped on a H_2 in a CY_3, and is dual to a $(2,2)$ SCFT. The solutions for generic x are dual to SCFTs with $(0,2)$ SCFT. It is natural to wonder if they lie within the class of explicit solutions found in [30]. If we perform the coordinate transformation

$$\begin{aligned} \sigma_3 &= \sigma_3' - d\tau' \\ d\tau &= d\tau' \end{aligned} \tag{B.8}$$

the metric (B.1) can be written

$$
\begin{aligned}
m^2 ds_{10}^2 = & \frac{4x}{(2x+1)^2}\bar{\Delta}^{1/2}\left[ds^2(AdS_3) + R_{(2)}^2 ds^2(\Sigma_l)\right] + \bar{\Delta}^{1/2}d\xi^2 + \frac{\cos^2\xi}{4\bar{\Delta}^{1/2}}d\Omega_2 \\
& + \frac{Z}{4\bar{\Delta}^{1/2}}\left[d\tau' - \frac{\cos^2\xi}{Z}\sigma_3' - \frac{8xR_{(2)}^2}{Z(2x+1)^2}(-\cos^2\xi + 2(2x-1)\sin^2\xi)W\right]^2 \\
& + \frac{x\cos^2\xi\sin^2\xi}{\bar{\Delta}^{1/2}Z}\left[\sigma_3' + lW\right]^2
\end{aligned}
\tag{B.9}
$$

where $Z = 4x\sin^2\xi + \cos^2\xi$. One can now compare with the solutions in [30] (see eq (2.11) of this reference).

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